COMPUTATIONAL MECHANICS
New Trends and Applications
E. Oñate and S. R. Idelsohn (Eds.)

THE LINEAR MODEL OF AN UNKNOWN DYNAMIC PROCESS

R. Ferreiro, J. Vidal & X. Pardo

Dep. Electrónica y Sistemas. Universidade da Coruña
E.S. Marina Civil, C/ Paseo de Ronda 51, 15011. A Coruña. SPAIN
Tel.: 34+81 167000 Fax.: 34+81 167101
e-mail: {ferreiro, vidal, pardo}@udc.es

Key words: Deterministic associative memory, Clustering, Mapping, Learning algorithm, Parameter identification.

Abstract. *This work describes a novel algorithmic approach to find the linear model of any dynamic process. Dynamic behaviour as a knowledge concept is acquired by means of proposed learning algorithm, being supported by DAM (deterministic associative memory) system. Depending on the particular use of process model, (system simulation, control design, reproduction of dynamic behaviour, parameter identification, plant diagnosis...) information to and from DAM will be stored and retrieved as demanded from a particular application by applying the proposed algorithmic methodology.*

1 INTRODUCTION

The principal benefits of learning systems, given the present state of its technological development, derive from the ability of learning algorithms to automatically synthesize mapping that can be used advantageously within a control system architecture. Examples of such mapping include a controller mapping that relates measured and desired plant outputs to an appropriate set of control actions or a model parameter mapping that relates the plant operating condition to an accurate set of model parameters (identification) (Walter L et al. 1992). In general, this mapping may represent dynamic functions.

Although there is some differences between adaptive and learning systems, we see that both adaptive and learning can be based on parameter adjustment algorithms, and that both make use of experimental information gained through closed-loop interactions with the plant. Clear differences exist between both methods: a control system that treats every distinct operating state as a novel one is limited to adaptive operation, whereas a system that correlates past experiences with past situations, and that can recall and exploit those past experiences, is capable of learning. In this work we will treat parameter estimation on the basis of a learning algorithm for unknown processes.

For a given model, linearization technique of substitution and binomial expansion can be quite tedious if the nonlinearities are numerous or involved. It may be easier to use a truncated Taylor's series expansion about the equilibrium condition. To be explicit, let us consider the equation

$$\ddot{X} + Y\dot{X} + X^2 + XY + \frac{\dot{Y}}{Z} + Y^2 Z = 0 \tag{1}$$

where all terms but one are nonlinear. Express $\ddot{X}$ as a function of the remaining dependent variables and their derivatives

$$\ddot{X} = -Y\dot{X} - X^2 - XY - \frac{\dot{Y}}{Z} - Y^2 Z \tag{2}$$

The truncated Taylor's series expansion is

$$\ddot{X} = \left.\frac{\partial \ddot{X}}{\partial \dot{X}}\right|_0 \dot{X} + \left.\frac{\partial \ddot{X}}{\partial X}\right|_0 X + \left.\frac{\partial \ddot{X}}{\partial \dot{Y}}\right|_0 \dot{Y} + \left.\frac{\partial \ddot{X}}{\partial Y}\right|_0 Y + \left.\frac{\partial \ddot{X}}{\partial Z}\right|_0 Z \tag{3}$$

Expressions for the partial derivatives can be obtained and can be evaluated at the equilibrium condition in the following way as:

$$\left.\frac{\partial \ddot{X}}{\partial \dot{X}}\right|_0 = -Y|_0 = -Y_0 \tag{4}$$

$$\left.\frac{\partial \ddot{X}}{\partial X}\right|_0 = -2X_0 - Y_0$$

$$\left.\frac{\partial \ddot{X}}{\partial \dot{Y}}\right|_0 = -\frac{1}{Z_0} \quad (Z_0 \textit{ cannot be zero})$$

$$\left.\frac{\partial \ddot{X}}{\partial Y}\right|_0 = -X_0 - 2Y_0 Z_0 \quad \textit{since} \quad \dot{X}_0 = 0$$

$$\left.\frac{\partial \ddot{X}}{\partial Z}\right|_0 = -(Y_0)^2 \quad \textit{since} \quad \dot{Y}_0 = 0$$

The linearized dynamic equation is

$$\ddot{x} + Y_0\dot{x} + (2X_0 + Y_0)x + \frac{\dot{y}}{Z_0} + (X_0 + 2Y_0 X_0)y + (Y_0)^2 z = 0 \tag{5}$$

Notice that the magnitudes of the coefficients of the linearized terms vary with the equilibrium conditions. This is a manifestation of the nonlinearities of the equation. In physical situations the expressions for the coefficients can often be simplified by relationships developed from the steady-state equation(s). Implicit in and essential to the technique of linearization about an equilibrium condition is the existence of an equilibrium condition and of all of the partial derivatives at the equilibrium condition. The partials may be zero but may not infinite thus the restriction above that Zo cannot be zero. Furthermore, the equations should be in the form of ordinary differential equations before linearization; this means that partial differentiation with respect to any of the dependent variables must be carried out before linearization to preclude losing any essential terms. Finally, how large a "small" perturbation can be before it introduces unacceptable errors is difficult if not impossible to determine prior to linearization; the errors are dependent upon the equations themselves and upon the specific equilibrium conditions of interest.

When equations or functional relationships are impossible or difficult to obtain, it may be possible to obtain linearized equations from operating data. The operating curves for an engine shown in figure 1 indicates that the output shaft speed *N* is a nonlinear function of both the fuel flow rate *Q* and the load torque *T*; i.e., *N* = *N(Q,T)*. Expanding this generalized function in a truncated Taylor's series expansion about the operating point 0 yields the linearized dynamic equation

$$n(t) = \left.\frac{\partial N}{\partial Q}\right|_0 q(t) + \left.\frac{\partial N}{\partial T}\right|_0 \hat{t}(t) \tag{6}$$

where *n*, *q* ant *t* are the perturbations of *N*, *Q* and *T* with respect to the operating point. The partial derivatives can be evaluated by determining the incremental change in *N* for incremental changes in *Q* and *T* alone; i.e.

$$\left.\frac{\partial N}{\partial Q}\right|_0 = \left.\frac{\Delta N}{\Delta Q}\right|_{T=T_0} = C_Q \quad and \quad \left.\frac{\partial N}{\partial T}\right|_0 = \left.\frac{\Delta N}{\Delta T}\right|_{Q=Q_0} = C_T \tag{7}$$

Thus, the dynamic behaviour of the engine may be approximated by the linear differential equation with constant coefficients

$$n(t) = C_Q q(t) + C_T \hat{t}(t) \tag{8}$$

C_0 and C_T should be re-evaluated if the operating point 0 changes significantly. There is not explicit steady-state equation; values of N_0, Q_0 and T_0 must be obtained directly from the operating data

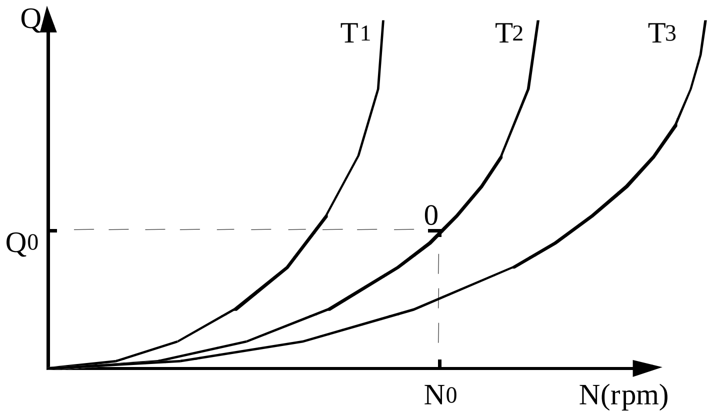

Fig. 1 Engine operating curves

The proposed linearization method achieves the same results under an automated experimental computation procedure.

The main steps in developing the linear model of a dynamic process is summarized as:

I. Finding the model order
II. Clustering definition into an hyper-cube rule base
III. Mapping the dynamic behaviour from experimental data
IV. Linear model achieving.

2 THE MODEL ORDER

An interesting feature of functional approximation algorithms is that they provide two generic classes of functions: linear and nonlinear functions. This is used in the following ways when modelling nonlinear dynamical systems[1,2]. Consider, for example, the linear model defined by the phase state space variable method

$$\frac{dx}{dt} = Ax + BuY = Cx \tag{9}$$

Expression (1) and (2) can be replaced with

$$\frac{dx}{dt} = f(x,u) \quad ; \quad Y = g(x,u) \tag{10}$$

where the functions *f* and *g* represents the system dynamics, *x* is the state vector and *u* is the excitation vector.

The expression for a general description for last definition by means of ordinary differential equations for a SISO case is:

$$B_m D^m u + B_{m-1} D^{m-1} u + \ldots + B_0 u = A_n D^n y + A_{n-1} D^{n-1} y + \ldots + A_0 y \quad with\, m < n \tag{11}$$

Persistent excitation of equation (11) by a unit step function such that

$$u(t) = \begin{pmatrix} 0, & t < 0 \\ 1, & t > 0 \end{pmatrix} \tag{12}$$

originates a sucessive series of derivatives in its output variable *y*, that can be described by its geometric interpretation as

$$D^i y = \frac{D^{i-1} y - D^{i-1} y_{OLD}}{H} \tag{13}$$

where H is the numeric integration step size. For an initial sufficiently short instant, all its derivatives are positive. So that its higher order derivative can be expressed as

$$D^n y = \frac{D^{n-1} y - D^{n-1} y_{OLD}}{H} > 0 \tag{14}$$

Consequently it results straightforward in that

$$D^{n+1} y = \frac{D^n y - D^n y_{OLD}}{H} < 0 \tag{15}$$

As consequence of (14) it can be stated that the first successive derivative of higher order whose value is negative, indicates the order of the model incremented in a unit. It can be applicable to systems defined by ordinary differential equations of type described in (11).

Lemma 1

In all stable systems that can be described under ordinary differential equations, excited by a step function, derivatives exist if and only if $D^n y > 0$.

As consequence, the order is *n* if $D^{n+1} y < 0$

Proof:

Given a system defined by an ordinary differential equations as

$$M\dot{v} + Bv = F \tag{16}$$

It can be described by means of a linear differential operator as

$$F = M D^1 v + Bv \tag{17}$$

The first derivative is achieved straightforward as

$$D^1 v = \frac{F}{M} - \frac{B}{M} v \geq 0 \tag{18}$$

the second successive derivative by recursive iteration is

$$D^2 v = \frac{D^1 v - D^1 v_{OLD}}{H} \leq 0 \Leftarrow D^1 v \leq D^1 v_{OLD} \tag{19}$$

Consecuently, the equation order is n = 2-1 = 1.
The algorithmic procedure is shown at figure 2

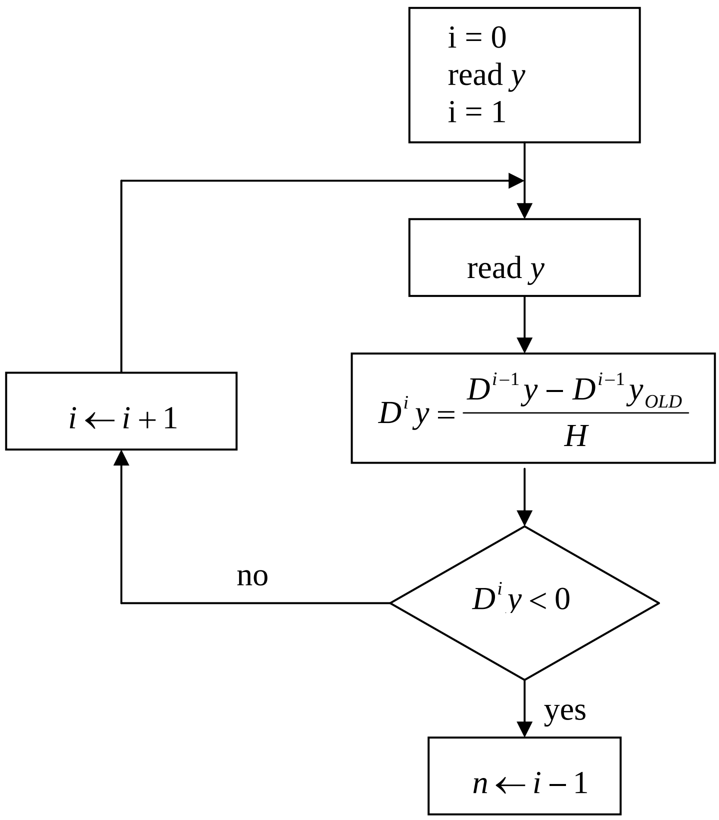

Fig. 2 Searching for the equation order

3 PRACTICAL APPLICATION

One of the most important and useful application in determining the system order is encountered in system identification tasks. Let's consider the case in which it is necessary to find the system model[3], that is the model parameters. Then, the process described in (16) is studied. Following the procedure shown from (17) to (19) it is concluded that the system order is 2. Consequently, the task to be developed in approaching the system parameters is carried out as described in[3]. Consequently, the structure of the differential equation under which the unknowns system must be described is shown as (16) and repeated here. So that

$$M\dot{v} + Bv = F \tag{20}$$

can be described by means of a linear differential operator as

$$F = MD^1 v + Bv \tag{21}$$

and conveniently rearranged for operational purposes as

$$aV_2{}' + bV_2 = V_1 \tag{22}$$

The higher order derivative of the system output variable $V_2{}'$ in a first order system, that is

$$V_2{}' = \left(1/a\right)V_1 + \left(1/b\right)V_2 \tag{23}$$

or in a normalized form,

$$V_2{}' = \left(k_1\right)V_1 + \left(k_2\right)V_2 \tag{24}$$

it could be expressed in matrix form as

$$V' = [V]\,K \tag{25}$$

and the plant parameters are directly achieved as,

$$K = [V]^{-1}\,V' \tag{26}$$

where K is the vector of plant parameters.

4 CONCLUSIONS

It has been shown that the approaching to a system order is a previous, basic and fundamental operation in system identification. So that, with a priori knowledge concerning the dynamic behaviour of an unknown system such as its evolution regarding its stability and external exciting forces, a good approach in determining its order is then achieved. The algorithmic simplicity and the clarity of methodology, makes this procedure a potential tool in system identification. Restrictions are encountered in determining when the unknown system is stable. So that this knowledge is essential for success

REFERENCES

[1] K.J.Astrom and Thomas J.McAvoy (1992). Ed. David A.White, Donald A.Sofge, *Handbook of Intelligent Control. Neural, Fuzzy and Adaptive Approaches*, Van Nostrand Reinhold. New York, Chap. 1, pp. 3-21

[2] Walter L. Baker and Jay A. Farrell (1992). Ed. David A.White, Donald A.Sofge, *Handbook of Intelligent Control. Neural, Fuzzy and Adaptive Approaches*, Van Nostrand Reinhold. New York, Chap. 2, pp. 35-61

[3] Ferreiro García, R (1994). *Associative memories as learning basis applied to the roll control of an aircraft*, 5th Int. Symp. AMST'94. Application of Multivariable System Techniques. University of Bradford. West Yorkshire, U.K.Hilda3